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Disorienting Dilemmas and Mathematical Reasoning: Exploring Transformative Learning in Preservice Teacher Education

CONSTANTA OLTEANU
Linnaeus University, Department of Mathematics

Abstract

Preservice elementary teachers often face challenges in solving arithmetic word problems, revealing gaps in conceptual understanding and reliance on procedural methods. This study examines how preservice teachers' reasoning relates to the emergence of disorienting dilemmas in written solutions to arithmetic word problems. By applying an analytical model that integrates reasoning attributes with the concepts of dilemmas and disorienting dilemmas, the study contributes to a more nuanced understanding of how cognitive and reflective processes are manifested in written mathematical work. Results from this study makes four key contributions: theoretically, by integrating disorienting dilemmas with a mathematics reasoning framework; empirically, by showing how specific reasoning attributes signal disorienting dilemmas; methodologically, by presenting a replicable four-step analysis model; and practically, by offering guidance for designing problems that support reflection and transformative learning in teacher education.

Keywords: Disorienting dilemmas, transformative learning, mathematical reasoning, preservice teachers, arithmetic word problems, teacher education

Introduction

Preservice teachers preparing to teach mathematics at the primary level often enter their teacher education programs with a wide range of experiences and backgrounds in mathematics. These experiences, shaped by their schooling, can include misconceptions, negative attitudes, and a reliance on rote-based or procedural learning methods (Beswick & Callingham, 2014; Jiang et al., 2022). While some preservice teachers may demonstrate solid foundational mathematical skills, many need significantly more confidence or conceptual understanding when solving mathematical problems. These challenges are particularly significant when the expectation is not only that the preservice teachers become proficient problem solvers but also to effectively guide their future students in developing strategies to solve several problems in mathematics using deeper mathematical reasoning. Problem-solving requires a comprehensive set of skills, particularly within the context of arithmetic word problems, which need to be used in teaching to allow future students to be better problem solvers. Preservice teachers in future work must be able to present and discuss multiple solution strategies, articulate and justify mathematical reasoning, and analyse mathematical content included in the problem formulation. These skills are essential for designing effective teaching sequences that improve students' problem-solving abilities. However, preservice teachers often use memorised algorithms and procedural understanding instead of conceptual understanding of mathematics (Berenger, 2018; Özgen & Alkan, 2014). Addressing these gaps requires more than simply changing content or problem structure. It necessitates transformative learning experiences that challenge and reshape preservice teachers' existing beliefs and assumptions about how mathematical problems are solved.

The research presented in this paper is located in transformative learning theory (see "Related research" section), paying particular attention to the first step in the process: the disorienting dilemma. Mezirow (1981) describes the disorienting dilemma as the moment when an individual "becomes critically aware of how their established habits of perception, thought, and action have shaped their understanding of a problem and their relationship to it." The concept will be more fully explored in the literature review. Specifically, this article addresses the following research question: How does preservice teachers' reasoning relate to the emergence of disorienting dilemmas in written solutions to arithmetic word problems?

Related research

Research on transformative learning often emphasises the process of change and the outcomes of learning experiences, focusing less on understanding the foundational dilemmas that trigger this transformation (DeAngelis, 2017, 2022). However, little attention has been paid to how such change processes unfold in the context of mathematics teacher education, particularly in terms of reasoning. Sletteboe (1997) identifies five key attributes that define *a dilemma* that can trigger transformative learning. The first attribute is involvement, engagement, or commitment—for a dilemma to lead to transformation, the individual must be emotionally or intellectually invested in the situation. Without this engagement, the dilemma may be overlooked. The second attribute occurs when the individual faces two or more equally undesirable or difficult choices, forcing them to confront complex issues. The uncertainty encourages more profound reflection on values and assumptions. The third attribute is awareness of alternatives—the individual must recognise different options to understand the situation as a dilemma. Without this awareness, critical decision-making cannot occur. The fourth attribute is the need for a choice—a dilemma involves a decision that must be made, convincing the individual to examine their beliefs and assumptions. The fifth attribute is uncertainty of action—a dilemma, as the individual is unsure of the course of action or its potential consequences. This uncertainty drives reflection and the search for new ways of thinking. The five attributes show that a dilemma can trigger transformative learning by challenging individuals to deeply engage with complex decisions that lead to personal growth and change.

When preservice teachers are confronted with dilemmas in mathematics education, they are often forced to deepen their mathematical reasoning and question their previous methods and assumptions. However, mathematics education research interprets mathematical reasoning in various ways (e.g., Olteanu, 2020). For instance, Lithner (2015) and Norqvist et. al. (2019) classifies mathematical reasoning into imitative and creative reasoning. These types of reasoning involve recalling solutions or using established procedures for problem-solving (imitative reasoning) and generating innovative solutions supported by logical arguments and mathematical concepts (creative reasoning). Study results indicate that working memory is essential in creative reasoning (Anjariyah et al., 2022) because it allows students to sift through relevant information and manage complex tasks. When working memory is limited, students may struggle to address non-routine problems and often resort to algorithmic reasoning instead of exploring more innovative approaches (Fyfe et al., 2019). Olteanu (2020) and Olteanu & Olteanu (2022) operationalised the reasoning concept by breaking it down into seven fundamental attributes: selecting, exploring, reconfiguring, filtering, encoding, abstracting, and connecting information.

Selecting involves the tools (e.g., diagrams, number lines) students choose to represent and solve problems. *Exploring* focuses on the strategies students employ to investigate the problem. A student might attempt multiple methods, such as guessing and checking, identifying patterns, tackling simpler problems, creating drawings, working backwards, and organising data in tables or lists. *Reconfiguring* focuses on restructuring the operation or text structure by breaking complex problems into simpler parts and thereafter reorganising data. *Filtering* involves distinguishing between relevant and irrelevant information, such as unnecessary numbers or context. *Encoding* examines how students translate information into mathematical representations. For example, accurately expressing a "10% discount" as a decimal (0.1) and using it in calculations shows an understanding of the problem's requirements and the ability to use mathematical language effectively. *Abstracting* evaluates whether students can generalise concepts or identify patterns that connect to broader mathematical ideas. A student solving a proportional reasoning problem who recognises that their approach can be applied to other contexts, such as scaling recipes or comparing ratios, demonstrates higher-order thinking and the ability to generalise knowledge. *Connecting* considers whether students link different parts of the problem or related concepts. For instance, a student solving a discount problem who relates percentages to fractions (e.g., recognising 10% as equivalent to 1/10) shows a deeper conceptual understanding by integrating multiple representations of the same idea.

Mezirow (1981) describes the disorienting dilemma as the moment when an individual becomes critically aware of how their established habits of perception, thought, and action have shaped their understanding of a problem and their relationship to it. However, as Mezirow notes, the disorienting dilemma is subjective; this means that what is disorienting for one person may not be for another. In this context, mathematical reasoning becomes essential to navigate disorienting dilemmas, improve preservice teachers' problem-solving skills and engage in deeper reflective reasoning. According to Mezirow (1978, p.108), disorienting dilemmas cannot be resolved by "simply acquiring more information, enhancing problem-solving skills or adding to one's competencies," but rather, through critical assessment of "how and why our habits of perception, thought and action have distorted the way we have defined the problem and ourselves in relationship to it" (Mezirow, 1991, p. 7).

DeAngelis (2022) defines a disorienting dilemma as the disjuncture between what one has known to be true and new ways of knowing. These fissures in ways of knowing offer learning and development opportunities (Mezirow, 1990). Preservice teachers can become more successful using disorienting dilemmas because disorienting dilemmas jolt adults into becoming curious and present (Johnson & Olanoff, 2020). In mathematics education, preservice teachers must learn not just the methods for solving problems but also how to engage with the reasoning processes to apply these methods for developing a more flexible approach to problem-solving that encourages exploring alternatives and a deeper understanding of mathematical concepts. However, this is a limited understanding of how preservice teachers' disorienting dilemmas emerge when mathematical reasoning is used in solving word problems. How mathematical reasoning can be used to design a disorienting dilemma that prompts a change in attitudes, beliefs, and values is not explicitly presented within the theory and is the subject of ongoing research (Code et al., 2022). By investigating the role of reasoning in the emergence of disorienting dilemmas, educators can better understand how to guide preservice teachers in their learning.

Theoretical framework

This study draws on three interconnected theoretical components—dilemmas, disorienting dilemmas, and reasoning attributes—to analyse preservice teachers' written solutions to arithmetic word problems and identify potential triggers of transformative learning.

Dilemmas and disorienting dilemmas in mathematical problem solving

Sletteboe (1997) defines a dilemma as a situation characterised by (1) engagement or commitment, (2) competing alternatives, (3) awareness of these alternatives, (4) the necessity of making a choice, and (5) uncertainty about the consequences. These characteristics imply cognitive or emotional tension that can prompt reflection and change. In mathematics education, dilemmas arise when preservice teachers must choose between different strategies or representations, often in the face of uncertainty. When such tension escalates and challenges prior assumptions, it may lead to a disorienting dilemma—a concept from transformative learning theory (Mezirow, 1981). A disorienting dilemma disrupts habitual patterns of reasoning and requires learners to assess their assumptions, ultimately leading to a deeper understanding critically.

Reasoning attributes

To understand how dilemmas emerge in problem-solving, the study employs a model of reasoning developed by Olteanu (2020), which includes seven attributes as shown in Figure 1:

Attributes of reasoning	Explanation
Selecting	Choosing tools, such as concrete, pictorial, or symbolic representations.
Exploring	Investigating concepts, counting methods, or problem-solving strategies.
Reconfiguring	Changing the structure or arrangement of an operation or a mathematical object.
Filtering	Choosing to include or exclude certain types of information.
Encoding	Representing ideas using words, phrases, or mathematical symbols.
Abstracting	Identifying common and essential features leads to creating new concepts.
Connecting	Emphasizing the connections and relationships between different pieces of content, concepts, or ideas.

Figure 1: *Reasoning attributes*

These attributes offer a lens for analysing the depth and complexity of students' engagement with a task and their capacity to shift from procedural to conceptual reasoning.

Integrating reasoning and dilemmas

This study draws on three interconnected theoretical components—dilemmas, disorienting dilemmas, and reasoning attributes—to analyse preservice teachers' written solutions to arithmetic word problems and identify potential triggers of transformative learning.

By combining these frameworks, the study investigates how preservice teachers' reasoning can signal the presence of dilemmas and, in some cases, disorienting dilemmas. For example, a student who answers a fraction problem with only a numerical solution (e.g., "8 – 3 = 5; 5/8") shows limited engagement, likely relying on encoding alone. This suggests no dilemma. However, if the student supplements the answer with a visual representation—such as a shaded pizza diagram—this may reflect uncertainty and an attempt to reconcile symbolic and conceptual reasoning. Multiple reasoning attributes (selecting, connecting, reconfiguring) are activated in such cases, indicating a potential dilemma. If the student experiences an internal conflict between competing strategies or begins questioning the adequacy of a purely procedural solution, the dilemma may become disorienting.

Thus, the framework links reasoning processes to transformative learning by identifying when and how dilemmas arise and intensify and provides a systematic method for interpreting written solutions that offer insight into the moments when preservice teachers begin to rethink established approaches and develop more flexible mathematical understanding.

Methodology

Research context and participants

This article presents a study on teacher education for preservice elementary teachers in Sweden. They enter teacher education programs through various routes: some enrol directly after completing upper secondary education, while others join with prior university studies or work experience. As a result, their ages, educational backgrounds, expectations for their studies, and perspectives on the teaching profession differ. The empirical studies in this article were conducted at a Swedish university with 37 preservice elementary teachers preparing to teach grades 4–6, where students are typically between 10 and 12 years old. Compulsory schooling in the Swedish education system covers grades 1–9 (ages 7–15), with grades 1–6 classified as elementary school. Elementary teachers are trained as generalists capable of teaching multiple subjects, although they often lack specialisation in mathematics. The teacher education programs include two 15 ECTS mathematics courses: one in the first year and another in the third year. The first course focuses on, among other things, deepening mathematical knowledge and developing problem-solving abilities related to various content areas in mathematics. Preservice teachers are expected to solve problems, present solutions, engage in discussions and conduct didactical analyses to plan effective teaching sequences. All participants provided informed consent, and ethical approval was obtained in accordance with Swedish Research Council's book, *God forskningssed* (Vetenskapsrådet, 2024) guidelines.

Data collection and analysis

Data were collected when preservice teachers solved arithmetic word problems in a written exam that is mandatory for the course (see below), using clear reasoning without algebraic equations. The problems were chosen because they require multiple steps, demand flexible reasoning, and typically provoke uncertainty or strategic shifts among students—conditions under which dilemmas are likely to emerge. Participants were asked to solve the problems and to explain their reasoning in writing, including any visual representations or alternative methods considered. The task prompts were intended to encourage reflective engagement with the mathematical content.

1. Planting flowers
Lena is planting flowers in her garden over four days as follows:
On the first day, she plants one-third of the flowers in the garden.
On the second day, she plants half of the remaining flowers.
On the third day, she planted half of the flowers still left.
On the fourth day, she plants the final 24 flowers.
Determine how many flowers Lena had at the start. Explain your reasoning using clear mathematical steps and illustrations without using equations.

2. Tomatoes and cucumbers
A school purchased 170 kg of tomatoes and cucumbers. The total cost for 170 kg is 1,200 SEK (Swedish Krona), and the price per kg is 6 SEK and 8 SEK, respectively. Determine the number of kilograms of tomatoes and cucumbers purchased.
Solve the problem step by step, clearly explaining each calculation without using equations.

3. The age difference
The mamma is 20 years older than her daughter. Four years ago, the mamma's age was three times that of her daughter. How old are both the mamma and the daughter? Use appropriate forms of representation and mathematical calculations to solve the problem without equations.

The analysis followed a combined qualitative and quantitative approach, guided by content analysis (Schreier, 2012) and framed by an integrated theoretical model combining the concepts of dilemma, disorienting dilemma, and reasoning attributes (Olteanu, 2020). The aim was to explore preservice teachers' written solutions to arithmetic word problems in order to identify cognitive and reflective processes that may support or hinder transformative learning. The analysis was conducted in four systematic steps.

Step 1: Initial reading and segmentation

All preservice teachers' solutions were initially read in full to gain a holistic understanding and were then segmented into meaning units, including verbal, symbolic, or visual expressions, based on shifts in reasoning or problem-solving approach. The goal of this step, in relation to the research question, was to prepare the data for analysis and facilitate the examination of how preservice teachers approach arithmetic problems and help reveal reasoning patterns that could be linked to dilemmas.

Step 2: Coding of reasoning attributes (Qualitative and quantitative analysis)

Each segment was coded using a predefined coding frame based on seven reasoning attributes developed from earlier research (Olteanu, 2020). This coding process combined qualitative interpretation with quantitative recording. It allowed for identifying patterns in how preservice teachers approached problem-solving and provided insight into how specific reasoning processes were connected to the emergence of dilemmas.

Step 3: Identification of dilemmas and non-dilemmas (Qualitative and quantitative analysis)

Five criteria, inspired by Sletteboe (1997), were applied to identify dilemmas in preservice teachers' written solutions to arithmetic word problems:

- Involvement or engagement in the problem was inferred from the preservice teachers who go beyond a single-step or purely procedural solution. Written signs of active engagement included the use of supplementary representations (e.g., diagrams), multiple attempts, or reflections embedded in the solution. Lack of elaboration or reliance on a single method suggested an absence of engagement with the problem as a dilemma.
- The presence of equally unsatisfactory alternatives was identified when the preservice teacher explored more than one strategy or representation without clearly favouring one, or when each attempt seemed incomplete or problematic. For example, the use of both a procedural calculation and a visual model suggests uncertainty about which strategy better conveys the concept.
- Awareness of alternatives was coded when a preservice teacher demonstrated recognition of more than one possible solution path. This became evident through visible strategy shifts, the use of multiple representations, or reconsiderations within the written work. Without signs of such awareness, the problem was unlikely to be experienced as a dilemma.
- A need for choice was interpreted from moments where the student appeared to make a decision between competing strategies. This could be seen in the selection of a final representation after initially trying another, or in the removal and replacement of earlier attempts. This decision-making process pointed to a resolution effort within the dilemma.
- Uncertainty about action was inferred from unfinished reasoning, expressions of doubt, or inconsistent justification. Multiple revisions, corrections, or unresolved explanations indicated that the preservice teacher was unsure of the best course of action or the sufficiency of their answer.

The use of these five criteria enabled a systematic identification of dilemmas within preservice teachers' written work. By combining qualitative interpretation of the written responses with binary coding (1 = dilemma present, 0 = not present), it was possible to trace where preservice teachers experienced disruptions in their reasoning. These disruptions marked potential moments of reflection and learning, offering insights into how mathematical understanding might shift through the confrontation of dilemmas.

Step 4: Identification of types of dilemmas (Qualitative and quantitative analysis)

The final step of the analysis compared patterns across preservice teachers' responses to explore how specific reasoning attributes were linked to dilemmas. Two types of dilemmas that fall under the broader category of dilemmas were identified: routine and disorienting dilemmas. Routine dilemmas involve choices between familiar strategies, often marked by hesitation or switching between known methods, but do not lead to more profound cognitive shifts. In contrast, disorienting dilemmas—central to transformative learning (Mezirow, 1991)—involve a disruption of prior assumptions and a shift from procedural to conceptual reasoning. This step also explored how reasoning behaviours (e.g., premature encoding or shallow exploration) co-occurred with specific dilemma types. By comparing preservice teachers' solutions, the analysis identified qualitative examples and quantitative patterns that give insights into how different reasoning attributes contribute to the emergence and nature of dilemmas, directly addressing the research question.

A reliability analysis (Cronbach's alpha) was used to ensure the internal consistency of the coding scheme used to analyse the preservice teachers' written solutions. The results show a high internal consistency for step 2, with a Cronbach's alpha value of 0.974. For step 3, the results showed good internal consistency, with a raw alpha value of 0.857, demonstrating high reliability. In step 4, the analysis revealed a high reliability of 0.857 concerning routine dilemmas, while a moderate reliability of 0.679 was found for disorienting dilemmas. The reliability analysis confirms that the coding scheme demonstrates high internal consistency for most steps, with slight variations in reliability for different types of dilemmas.

Results

This section presents the findings from the analysis of preservice teachers' written solutions to three arithmetic word problems. The results are organised in two parts. First, it presents an overview of how dilemmas were distributed across the dataset, including how they were classified as either routine or disorienting and how these types relate to specific reasoning attributes. Second, the qualitative analysis shows how the dilemmas developed and whether they were resolved using familiar strategies or became disorienting, prompting a shift in perspective or more profound reflection.

Distribution of dilemmas

Figure 2 displays the number of participants who exhibited each attribute, the type of dilemma associated with it (None, Routine, or Disorienting), the outcome distribution regarding successful, partially successful, or unsuccessful solutions, and examples illustrating how dilemma-related reasoning emerged. Four preservice teachers did not solve any of the problems and thus were not associated with any specific reasoning attributes or dilemma types within the scope of this analysis. Among the remaining participants, variation was observed in the use of reasoning attributes. Some attributes, such as selecting and exploring, were linked to routine and disorienting dilemmas, while others, such as abstracting and connecting, were exclusively associated with disorienting dilemmas. These findings highlight how different cognitive and reflective processes can support or hinder successful problem-solving, depending on how students navigate the dilemmas that arise in their reasoning.

Reasoning attribute	No. of participants	Dilemma type	Outcome distribution (Success / Partial / Unsuccessful)	Examples of dilemma-related reasoning
Selecting	33	None (6) Routine (12) Disorienting (15)	None: 5/1/0 Routine: 5/5/2 Disorienting: 11/3/1	Choosing between representations; disorienting dilemmas triggered by uncertainty in choice.
Exploring	33	Routine (17) Disorienting (16)	Routine: 8/6/3 Disorienting: 12/3/1	Exploring strategies led to routine dilemmas when unproductive paths were chosen; disorienting when deeper conflicts arose.
Reconfiguring	22	Routine (11) Disorienting (11)	Routine: 6/3/2 Disorienting: 8/2/1	Re-evaluating and restructuring info led to dilemmas in both expected and unexpected ways.
Filtering	17	None (8) Routine (9)	None: 6/2/0 Routine: 5/2/2	Automatic filtering sometimes overlooked key info, leading to routine dilemmas.
Encoding	15	Routine (15)	Routine: 8/5/2	Routine dilemmas due to misinterpretation when translating natural language to mathematics.
Abstracting	11	Disorienting (11)	Disorienting: 9/1/1	Abstracting triggered dilemmas when unfamiliar patterns appeared.
Connecting	11	Disorienting (11)	Disorienting: 8/2/1	Connecting representations led to disorientation due to incoherence.
Combination: Selecting + Exploring	33	Disorienting (33)	Disorienting: 25/6/2	Combined use deepened dilemmas when strategy misaligned with the problem.
Combination: Abstracting + Connecting	11	Disorienting (11)	Disorienting: 9/1/1	Strongly associated with cognitive conflict and insight; transformative potential.

Figure 2: Distribution and characteristics of dilemmas with reasoning attributes

Qualitative content analysis

Absence of dilemma: Routine reasoning without disruption

In several instances, preservice teachers solved the arithmetic word problems without encountering any dilemma, particularly about the reasoning attributes of selecting and filtering. These cases were characterised by fluent and unproblematic reasoning, where the choice of tools, representations, or strategies appeared intuitive and straightforward. In problems such as planting flowers (Figure 3), a subset of preservice teachers immediately identified appropriate representations (e.g., number lines or diagrams) and applied them effectively without hesitation or ambiguity.

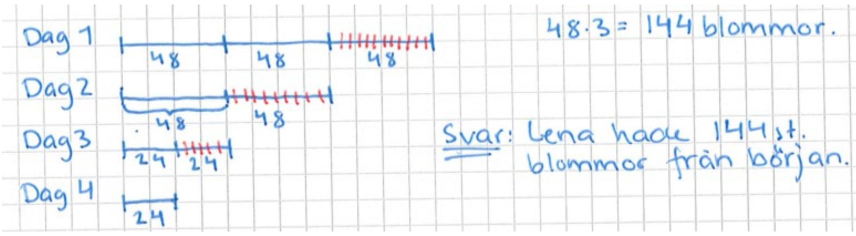

Figure 3: Absence of dilemma – planting flowers

The preservice teacher's visualisations aligned with the structure of the problem, and no signs of uncertainty or confusion were evident. The number lines or diagrams served their intended purpose — to organise information and guide step-by-step calculations — with no signs of breakdown or misalignment. Similarly, when reasoning involved filtering, some students clearly could isolate the most relevant quantities and constraints (e.g., price per kilogram, total weight, or fixed age differences) while discarding irrelevant narrative details. Encoding information into mathematical symbols or expressions was accurate and purposeful, and no evidence of representational conflict or strategic indecision emerged. These examples underscore that not all mathematical problem-solving is marked by visible dilemmas. Some preservice teachers moved directly and effectively from problem interpretation to solution without facing conceptual or procedural barriers. Such instances were coded as "None" (Figure 2) in the dilemma type and typically resulted in successful or partially successful outcomes.

Routine Dilemmas: Navigating familiar territory

Routine dilemmas involve problems that can be systematically broken down into smaller, manageable steps using known mathematical tools and strategies. These dilemmas typically require preservice teachers to apply concepts such as fractions, arithmetic operations, and proportional reasoning. Overcoming the dilemmas depends on selecting the tools, developing effective strategies, and focusing on essential information.

Selecting tools

In the planting flowers and the age difference problems, the most common preservice teachers are tasked with calculating how many flowers were planted over several days, each involving a fractional portion of the total. For example, preservice teachers represent planting one-third of the flowers on Day 1 using a diagram or a number line (Figure 4). These tools help preservice teachers track how each fraction contributes to the total. Similarly, in the age difference problem, preservice teachers determine the current ages of the mamma and daughter, given a 20-year difference and a condition from four years ago when the mamma's age was three times the daughter's. For instance, preservice teachers use tools such as age diagrams and timelines to help them represent these relationships visually, enabling them to apply basic arithmetic to find the solution (Figure 5).

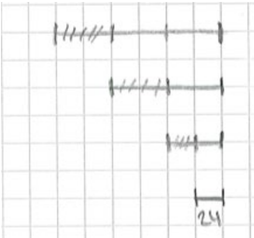

Figure 4: Planting flowers - visualisation

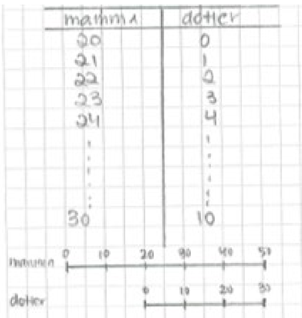

Figure 5: Age difference - visualisation

The tomato and cucumber problem challenges preservice teachers to distribute 170 kg of produce, priced differently, to meet a total cost of 1,200 SEK. This time, some preservice teachers immediately begin performing calculations to determine how many kilograms of each vegetable were purchased or using trial and error with weights (Figure 6).

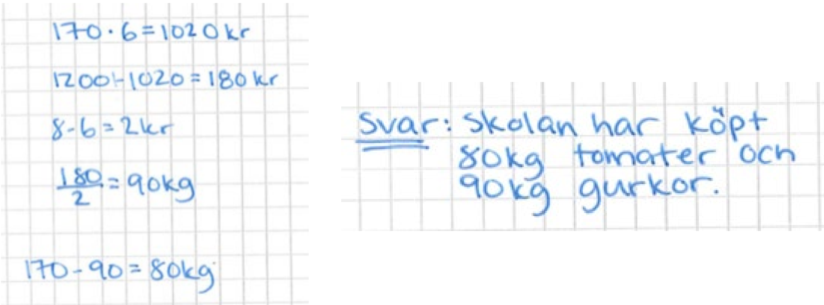

Figure 6: Tomatoes and cucumbers – selected tools

Developing strategies

Routine dilemmas also require preservice teachers to explore and refine their problem-solving strategies. In the planting flowers problem, the preservice teacher works backwards from the total number of flowers planted on the final day to determine how many were planted each day, using a number line to illustrate the relationship between the parts and the whole (Figure 7).

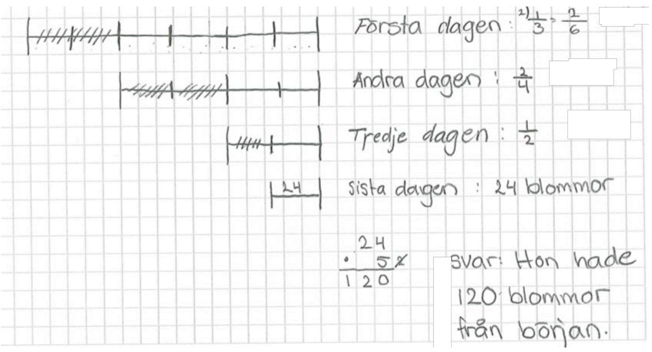

Figure 7: Planting flowers – backwards strategy

In the tomato and cucumber problem, preservice teachers test various distributions of weight and cost, adjusting their approach to align with the constraints. For instance, Figure 6 shows that most students solve the problem by calculating if all 170 kg were tomatoes, priced at 6 SEK per kilogram.

This results in a cost of 1,020 SEK, less than the total cost. The difference between the total cost (1,200 SEK) and this amount (1,020 SEK) is 180 SEK, representing the portion of the cost attributed to cucumbers. Next, the price difference between cucumbers (8 SEK/kg) and tomatoes (6 SEK/kg) is calculated as 2 SEK per kilogram. Dividing the remaining 180 SEK by this price difference (2 SEK/kg) gives 90 kg of cucumbers purchased. Finally, subtracting this 90 kg from the total weight (170 kg) gives 80 kg of tomatoes. In conclusion, the school purchased 80 kg of tomatoes and 90 kg of cucumbers.

In the age difference problem, preservice teachers often begin by calculating the current age of the mamma and then test age combinations to verify if they satisfy the past relationship (Figure 8). At the top, the relationship between the mamma and daughter's ages is represented visually and numerically. Breaking the problem into these smaller steps allows them to solve it more systematically.

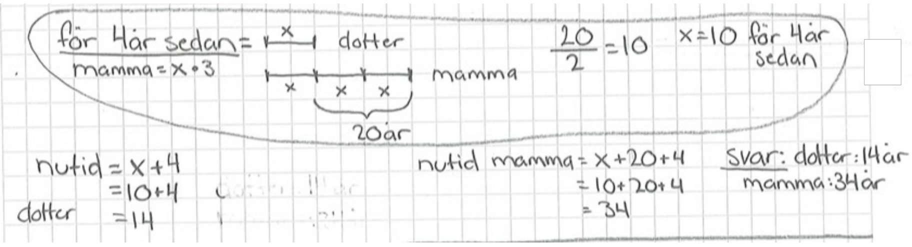

Figure 8: Age difference - strategy

Filtering and encoding information

Filtering and encoding essential details using mathematical symbols are critical skills in routine dilemmas. In the planting flowers problem, preservice teachers focus on fractional amounts and how they sum up to the total, ignoring extraneous details. Symbols and arithmetic operations help them track their reasoning. For the tomato and cucumber problem, preservice teachers concentrate on the total weight, total cost, and price per kilogram, using these elements to guide their calculations. Similarly, in the age difference problem, they focus on the 20-year age difference and the tripling condition from four years ago, abstracting these into a structured arithmetic expression to find the correct ages.

Disorienting dilemmas: Confronting disruptions in mathematical reasoning

Disorienting dilemmas emerge when preservice teachers face disruptions in their problem-solving processes that interfere with the coherence and depth of their mathematical reasoning. These dilemmas are not simply mistakes but signal deeper cognitive challenges, such as confusion, uncertainty, or breakdowns in structuring and interpreting the task. The analysis identifies three key categories through which disorienting dilemmas manifest.

Breakdown in representational coherence

In the planting flowers problem, some preservice teachers produce imprecise or incorrect diagrams when attempting to represent how fractions accumulate over several days. Rather than clarifying the relationships between parts and the whole, these diagrams obscure them, leading to miscalculations of how many flowers are planted daily (Figure 9). The representational breakdown suggests a lack of alignment between visual tools and mathematical reasoning.

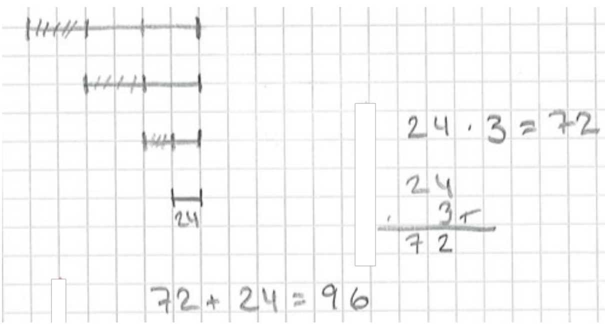

Figure 9: Planting flowers - tool selection as disorienting dilemma

Similarly, in the age difference problem, preservice teachers sometimes draw timelines or use arithmetic notations that do not accurately capture the age relationships, particularly the “three times as old” condition from four years ago. These flawed representations confuse with preservice teachers misreading the relational structure of the ages. Representational coherence breaks down in the tomato and cucumber problem when preservice teachers ignore or fail to map the proportional relationship between price and weight. Some use arbitrary numbers or inconsistent labelling in their tables or equations, which leads to incoherent or contradictory calculations.

Figure 10 presents a preservice teacher’s solution as an example of a successful solution and a disorienting dilemma. Initially, the preservice teacher appears disoriented by the contradictory nature of the problem conditions. This disorientation is evident in the decision to test various representations (bar models, a systematic table of numerical trials, and verbal explanations) values and compare outcomes.

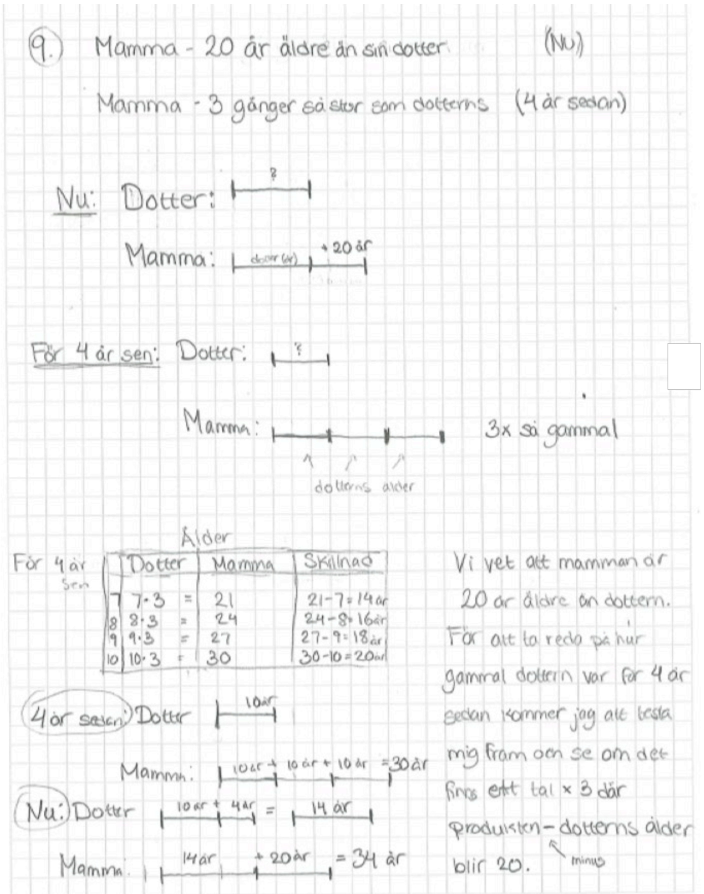

Figure 10: Age difference - disorienting dilemma

The preservice teacher does not proceed through algebraic formalism but rather through a reasoning strategy rooted in trial and error supported by visual representations. This exploratory process reveals the preservice teacher's engagement with a disorienting dilemma reconciling two interrelated but distinct temporal relationships (difference and ratio) into a coherent solution. The preservice teacher finds a valid solution, determining that the daughter is currently 14 years old and the mother 34 while validating that four years ago, the age ratio was 30:10, or 3:1. Several reasoning attributes are activated in the process, including selecting (appropriate visual and tabular methods), exploring (trying different numerical options), reconfiguring (switching between representations), filtering (focusing on relevant data), and connecting (linking age difference with proportional reasoning). From a teacher education perspective, this episode demonstrates how disorienting dilemmas can function as productive cognitive events that challenge preservice teachers to reflect deeply and integrate multiple reasoning strategies. Rather than viewing the disorientation as an obstacle, it becomes a catalyst for reflective and transformative thinking, aligned with the development of rigorous mathematical thinking.

Fragmented reasoning and unstructured exploration

This pattern is evident when preservice teachers engage only partially with the problem structure, resulting in disjointed or incomplete reasoning chains. In the planting flowers problem, some preservice teachers calculate how many flowers were planted daily but do not connect this to the whole sequence across all days. Their reasoning lacks a cumulative perspective and fails to verify whether all fractional parts add up to the total. In the age difference problem, some preservice teachers guess ages based on one part of the condition (e.g., 20-year difference) but do not test whether their answer fits the second condition (e.g., "three times as old" four years ago). The solution is treated as a one-step guess rather than a coordinated reasoning process. In the tomato and cucumber problem, many preservice teachers try out random combinations of weights and prices without isolating key variables or using a systematic approach. This unstructured exploration rarely leads to a valid solution and shows a lack of integration of the problem's constraint (Figure 11).

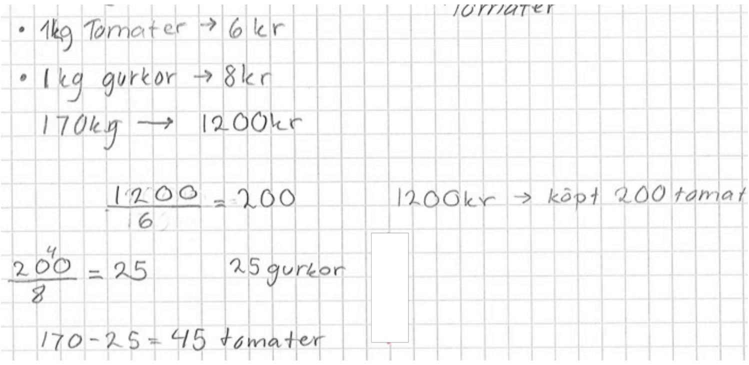

Figure 11: Tomatoes and cucumbers - exploration as disorienting dilemma

This solution also demonstrates the role of working memory in supporting reasoning. The student moves beyond simple algorithms and engages in flexible, multi-step thinking. Working memory helps students filter relevant information and manage and coordinate several numerical relationships across steps, but the last calculation is wrong.

Cognitive rigidity and failure to restructure

This category captures instances where preservice teachers persist in flawed methods despite signs that their approach is ineffective. In the planting flowers problem, preservice teachers divide the total number of flowers into daily fractions. Still, when early steps fail or yield inconsistencies, they continue with the same method without adjusting their strategy or reconsidering their representation. In the tomato and cucumber problem, cognitive rigidity is evident when students fixate on one trial-and-error method without stepping back to analyse the structure of the cost-weight relationship. Even when results do not add up, they fail to pivot to a new strategy. In the age difference problem, some students apply arithmetic operations rigidly (e.g., subtracting or multiplying ages) without considering whether the operation fits the problem logic. They do not attempt to reframe the relationships into simpler steps or build a coordinated model of the age conditions.

Discussion

This study investigates how preservice teachers' reasoning relates to the emergence of disorienting dilemmas in their written solutions to arithmetic word problems. By applying an analytical model that integrates reasoning attributes with the concepts of dilemmas and disorienting dilemmas, the study contributes to a more nuanced understanding of how cognitive and reflective processes are manifested in written mathematical work.

The findings show that preservice teachers' reasoning plays a central role in the emergence of dilemmas, particularly disorienting dilemmas. Written solutions demonstrating multiple reasoning attributes (e.g., selecting visual tools, reconfiguring the problem structure, or connecting different representations) are likelier to exhibit signs of a dilemma and include shifts between representations or the juxtaposition of alternative strategies. A dilemma becomes disorienting when such shifts are accompanied by conceptual tension or dissatisfaction with prior strategy. This interpretation is supported by Mezirow's (1981, 1991) assertion that disorienting dilemmas arise when existing knowledge or strategies are disrupted. However, this study extends Mezirow's theory by empirically linking specific reasoning attributes, such as reconfiguring and abstracting, to disorienting dilemmas. Whereas prior research has primarily focused on transformative learning outcomes (DeAngelis, 2017, 2022), this study addresses a significant gap by identifying triggers that may initiate such change by examining preservice teachers' written responses and the attributes of reasoning that serve as potential catalysts for disorienting dilemmas in mathematics classrooms. Sletteboe's (1997) framework was instrumental in distinguishing the presents of reasoning attributes in dilemmas and disorienting dilemmas. While earlier studies (e.g., Lithner, 2015; Norqvist, et. al., 2019) have differentiated between imitative and creative reasoning, this study adds a transformative dimension. It shows that certain instances of creative reasoning, when associated with uncertainty and conceptual reconfiguration, are innovative and disorienting capable of catalysing transformative learning. Moreover, by defining reasoning through seven attributes (Olteanu, 2020), the study offers a practical tool for identifying how preservice teachers engage in mathematical thinking and how working memory supports this process. Working memory allows students to focus on relevant numerical information and maintain attention throughout complex, multi-step tasks (Anjariyah et al., 2022). For instance, solutions based only on encoding (symbolic representation) may show procedural skill but little reflection. In contrast, solutions involving abstracting and connecting reflect a deeper understanding of the problem's structure. These reasoning patterns help identify moments where students may be open to reflection and transformation. As Fyfe et al. (2019) observe, students with limited working memory tend to rely on routine methods. However, in this study, the preservice teachers go beyond algorithms, engaging thoughtfully with disorienting dilemmas.

Routine dilemmas arise when preservice teachers rely on familiar problem-solving strategies and tools. Such dilemmas are evident in problems where visual representations (e.g., number lines or bar diagrams) and basic arithmetic operations are effectively used, as seen in the "Planting flowers" and "Tomatoes and cucumbers" problems. These cases align with earlier research showing that preservice teachers draw on procedural knowledge and established techniques when faced with standard tasks (Berenger, 2018; Özgen & Alkan, 2014). Routine dilemmas often involve imitative reasoning (Lithner, 2015; Norqvist, et. al., 2019) because preservice teachers follow well-practiced procedures without questioning their appropriateness or considering alternative strategies. While such dilemmas can reinforce procedural fluency, they may limit the development of more flexible and creative problem-solving skills. This study highlights the importance of reasoning attributes (Olteanu, 2020) to support preservice teachers in, for instance, choosing appropriate tools and focusing on relevant information. However, when applied in isolation, these attributes may not foster the reflective thinking needed to address more complex mathematical challenges.

Disorienting dilemmas emerge when preservice teachers face challenges that disrupt their established ways of thinking, typically when familiar strategies fail, or the task structure challenges their expectations. A clear example is the age difference problem, in which preservice teachers struggle to make sense of the relationship between fathers' and daughters' ages, leading to uncertainty and cognitive tension, which act as catalysts for change according to transformative theory. Disorienting dilemmas require preservice teachers to go beyond routine methods and instead develop adaptable problem-solving approaches that align with the structure and logic of the problem. In the age difference problem, many preservice teachers had to reconsider their assumptions and modify their strategies to grasp the temporal relationship involved. This process of conceptual adjustment marks a shift from procedural to conceptual reasoning, a key transition in transformative learning.

Through this analysis, the study demonstrates how disorienting dilemmas can serve as indicators of potential transformation when identified through specific reasoning patterns. It also provides actionable insights for teacher educators by showing how arithmetic tasks and assessment strategies can be designed to support the emergence of such dilemmas and foster reflective, conceptually rich learning experiences.

Practical implications for teacher education

The study suggests that disorienting dilemmas may not always arise from the complexity of the problem itself but rather from preservice teachers' engagement during the problem-solving and having some knowledge of problem-solving strategies. For instance, in the tomatoes and cucumbers problem, some teachers attempted arbitrary weight distributions, which led to incomplete or incorrect solutions, which indicates that a disorienting dilemma can emerge when existing knowledge and approaches prove insufficient, prompting cognitive conflict and reflection. Consequently, mathematics education should include opportunities that move beyond repeated procedures and foster reflective, conceptual, and creative thinking regarding solving problems in mathematics. However, this also suggests that intentionally designing problems to trigger disorienting dilemmas requires carefully considering preservice teachers' prior knowledge. Teacher education programmes can develop a more flexible and conceptual understanding of mathematics problem solving, essential for effective classroom practice, by engaging preservice teachers in routine and disorienting dilemmas. Teacher education programmes can integrate tasks that intentionally introduce disorienting dilemmas to prompt preservice teachers to evaluate their problem-solving strategies critically. These tasks should prioritise conceptual understanding and promote reflective thinking. While grappling with such dilemmas may enhance preservice teachers' mathematical thinking, the broader pedagogical value lies in how these experiences translate into improved teaching practice. Through this process, preservice teachers may become better equipped to explain mathematical concepts more clearly, anticipate and diagnose common student misunderstandings, and design tasks that encourage flexible thinking in their classrooms. Engaging with disorienting dilemmas can help preservice teachers internalise content knowledge and ultimately support more effective teaching by enabling them to guide their students through similarly rich learning experiences.

Broader educational context

While this study focuses on arithmetic word problems, the content can be extended to other mathematical domains and classroom settings. For example, how do preservice teachers transfer the problem-solving skills developed through confronting dilemmas in controlled tasks to more dynamic classroom environments? In real classrooms, preservice teachers may face even more complex dilemmas involving multiple students and varying levels of mathematical understanding. Future research could explore how these dilemmas manifest in lectures and seminaries at the university level and how teacher educators can help preservice teachers navigate them.

Conclusion and contribution

This study makes several significant contributions to the field of mathematics education. First, it presents a theoretical contribution by offering a novel integration of Mezirow's concept of disorienting dilemmas with a reasoning framework grounded in mathematics education. This synthesis enables a more fine-grained analysis of how transformative learning may emerge within routine classroom activities by linking moments of disruption to specific patterns of student reasoning. Second, the study provides an empirical contribution. An analysis of preservice teachers' written solutions to arithmetic word problems offers concrete examples of how disorienting dilemmas can be inferred from students' reasoning processes. The findings show that certain combinations of reasoning attributes, particularly selecting, reconfiguring, and connecting, can signal moments of cognitive or epistemological conflict that may lead to reflection and conceptual change. Third, the study makes a methodological contribution by introducing a four-step analysis model that combines quantitative and qualitative content analysis (Schreier, 2012) with reasoning. This model offers a systematic and replicable approach to studying the first stage of transformative learning using written problem-solving solutions. Finally, the study offers a practical contribution to teacher education by suggesting that teachers can design arithmetic problems and assessment strategies more likely to trigger reflection and support transformation. By paying close attention to students' reasoning attributes, educators can better support learners as they encounter and work through disorienting but potentially productive dilemmas.

Recommendations for future research

Future research could expand on this study by investigating the role of disorienting dilemmas in other areas of mathematics education. Specifically, studies could explore how preservice teachers encounter dilemmas in more complex domains beyond arithmetic. Finally, longitudinal studies could assess the long-term impact of integrating reasoning attributes and disorienting dilemmas into teacher education programs, further refining pedagogical strategies to better support preservice teachers in becoming effective mathematics educators.

Limitations of the study

While this study provides valuable insights into how preservice teachers encounter and navigate routine and disorienting dilemmas in arithmetic problem-solving, several limitations should be considered when interpreting the findings. First, the sample size was small, limiting the results' generalisation and focusing primarily on arithmetic word problems, which limits the applicability of the results to other areas of mathematics. The dilemmas encountered in more complex mathematical domains, such as algebra or geometry, may differ, so further research should explore dilemmas in these areas to offer a broader perspective on the problem-solving experiences of preservice teachers across various mathematical content.

The study was also conducted over a short period, which needs to account for the long-term development of preservice teachers' problem-solving abilities. A longitudinal study would better understand how preservice teachers' strategies and reasoning evolve, particularly in response to routine and disorienting dilemmas. A longitudinal study helps capture the problem-solving development throughout preservice teacher education. Furthermore, the study did not explore the emotional experiences of preservice teachers (frustration, anxiety, or confidence), which can significantly influence how they engage with dilemmas and affect their problem-solving process. Future research could investigate the interaction between cognitive and emotional factors in problem-solving approaches.

Another limitation is the study's context; a specific teacher education program may have influenced how dilemmas were encountered and addressed. This program's instructional practices and curricular design may only represent some teacher education programs. Therefore, future studies comparing preservice teachers' experiences across different programs would help determine how pedagogy and curriculum variations impact problem-solving and mathematical reasoning development.

Author’s Note:
Professor Constanta Olteanu’s research encompasses a broad range of perspectives within mathematics education, consistently focusing on the connection between what is enacted in teaching and what becomes possible for students and preservice teachers to learn. By integrating post-structuralist philosophical perspectives with didactical theory, such as variation theory, her research strengthens the understanding of how mathematical content can be used in iterative processes by teachers and preservice teachers to improve students' abilities to observe, reason, and develop mathematical knowledge. Her work contributes to both a detailed exploration of mathematics learning and a deeper insight into the dynamic interaction between teaching and learning, as well as the integration of research evidence into the enactment of mathematics instruction.

AI Use Statement
No AI has been used in the generation of this text.

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